MTH 304
Topology **Homework 2**

1. Let A, B, and A_α denote subsets of a topological space X. Determine whether the following hold.
   (a) If $A \subseteq B$, then $\bar{A} \subseteq \bar{B}$.
   (b) $\overline{A \cup B} = \bar{A} \cup \bar{B}$
   (c) $\cup_\alpha \bar{A}_\alpha = \overline{\cup A_\alpha}$.
   (d) $\overline{A \cap B} = \bar{A} \cap \bar{B}$
   (e) $\cap_\alpha \bar{A}_\alpha = \overline{\cap A_\alpha}$.
   (f) $\overline{A \setminus B} = \bar{A} \setminus \bar{B}$.
   (g) $\overline{A} \times B = \bar{A} \times \bar{B}$.
2. Show that X is Hausdorff if, and only if the diagonal $\Delta = \{(x, x) : x \in X\}$ is closed in the product topology on $X \times X$.
3. Consider $\mathbb{R}$ with co-finite topology. Find the closure of the set $A = \{1/n : n \in \mathbb{N}\}$.
4. Exercises 16 and 18, Section 17, page 99.
5. For $A \subseteq X$, define the boundary of A by
$$\partial A = \bar{A} \cap \overline{(X \setminus A)}.$$
   (a) Show that the interior A^0 and boundary ∂A are disjoint.
   (b) Show that $\partial A =$ if, and only if A is both open and closed.
   (c) Show that U is open if, and only if $\partial A = \bar{U} \setminus U$.
   (d) If U is open, is it true that $U = (\bar{U})^0$? Justify your answer.